APPENDIX A

Fig. 10 is an example of the Quine-McCluskey method for logic minimization and should be known to those trained in the art. The method is used to reduce the number of minterms required to implement Boolean logic functions. We start with a truth table [900] representing the logic function to be minimized. The terms which cause the Boolean function to evaluate to true, can also be represented in sum-of-product form [950]. The Quine-McCluskey method begins by having the product terms written out in a table and grouped by the number of 1's in each term [910]. Terms are then compared between the adjacent groups and those which differ in only one bit position are combined with the differing bit replaced by a "don't care". For example terms "0010" and "0011" differ in the last bit position, so these terms are combined to form "001x" [920]. The terms are checked off as they are replaced. The result after the first pass is shown [930]. The process is repeated until no more terms can be combined [940]. The resulting unchecked terms form the "prime implicants" which are listed in the prime implicant table [950]. This table [950] is used to determine the minimal number of prime implicants needed to represent the original logic function. The resulting function can, again, be expressed in the sum-of-products form [960] using the minimal set of prime implicants. The method can be extended to handle "don't care" values in the original Boolean logic function as described by Fig. 11 below.

Fig. 11 works through an example of the Quine-McCluskey method for logic minimization where the Boolean logic function [980] includes "don't cares". The method proceeds by finding the prime implicants assuming that the "don't cares" evaluate to true [982] and [984]. However, when finding the minimum number of minterms needed to implement the function, the "don't care" terms are not included [986]. The minimized function may able to be expressed using less terms [990]. In this case if the "don't cares" were not allowed, the function would minimized to $f = BC\overline{D} + A\overline{B}C$, however allowing for the "don't cares" reduces the function to $f = BC\overline{D} + A\overline{B}$. Obviously allowing "don't cares" becomes more and more beneficial as the number of terms in the logic function increases.

Truth Table	
0000	0
0001	0
0010	1
0011	1
0100	1
0101	1
0110	0
0111	1
1000	0
1001	0
1010	0
1011	0
1100	1
1101	1
1110	0
1111	1

900

1's	Terms	Pass 1	Pass 2
1	0010		
	0100		
2	0011		
	0101		
	1100		
3	0111		
	1101		
4	1111		

910

920

1's	Terms	Pass 1	Pass 2
1	0010 ✓	001x	
	0100		
2	0011 ✓		
	0101		
	1100		
3	0111		
	1101		
4	1111		

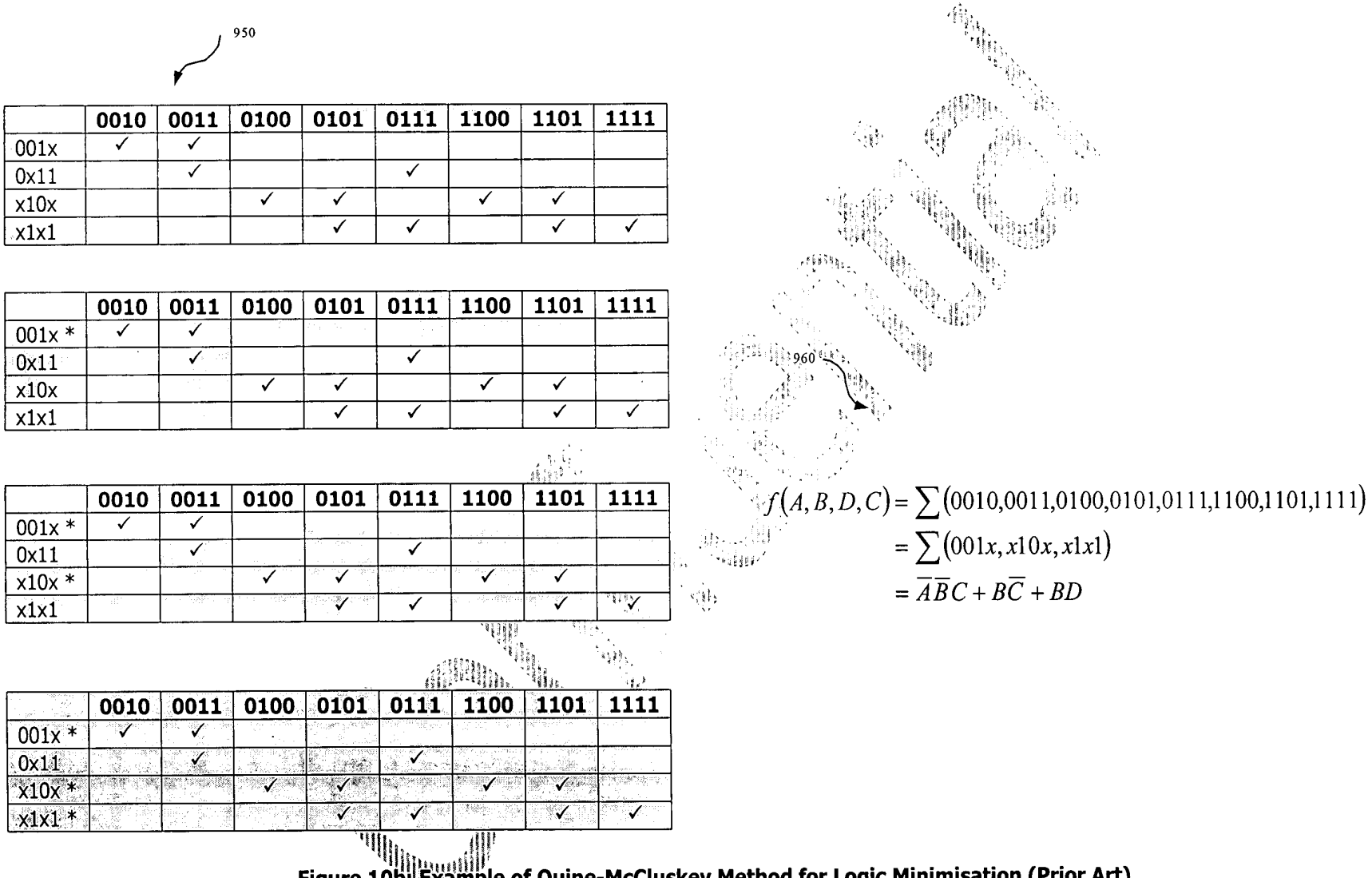
930

1's	Terms	Pass 1	Pass 2
1	0010 ✓	001x	
	0100 ✓	010x	
2	0011 ✓	x100	
	0101 ✓	0x11	
	1100 ✓	01x1	
3	0111 ✓	x101	
	1101 ✓	110x	
4	1111 ✓	x111	
		11x1	

940

1's	Terms	Pass 1	Pass 2
1	0010 ✓	001x	x10x
	0100 ✓	010x ✓	x1x1
2	0011 ✓	x100 ✓	
	0101 ✓	0x11	
	1100 ✓	01x1 ✓	
3	0111 ✓	x101 ✓	
	1101 ✓	110x ✓	
4	1111 ✓	x111 ✓	
		11x1 ✓	

Figure 10a: Example of Quine-McCluskey Method for Logic Minimisation (Prior Art).



	0010	0011	0100	0101	0111	1100	1101	1111
001x	✓	✓						
0x11		✓			✓			
x10x			✓	✓		✓	✓	
x1x1				✓	✓		✓	✓

	0010	0011	0100	0101	0111	1100	1101	1111
001x *	✓	✓						
0x11		✓			✓			
x10x			✓	✓		✓	✓	
x1x1				✓	✓		✓	✓

	0010	0011	0100	0101	0111	1100	1101	1111
001x *	✓	✓						
0x11		✓			✓			
x10x *			✓	✓		✓	✓	
x1x1				✓	✓		✓	✓

	0010	0011	0100	0101	0111	1100	1101	1111
001x *	✓	✓						
0x11		✓			✓			
x10x *			✓	✓		✓	✓	
x1x1 *				✓	✓		✓	✓

$$f(A,B,D,C) = \sum(0010,0011,0100,0101,0111,1100,1101,1111)$$
$$= \sum(001x, x10x, x1x1)$$
$$= \overline{A}\,\overline{B}C + B\overline{C} + BD$$

Figure 10b: Example of Quine-McCluskey Method for Logic Minimisation (Prior Art).

Truth Table	
0000	x
0001	x
0010	0
0011	0
0100	0
0101	0
0110	1
0111	0
1000	x
1001	x
1010	1
1011	1
1100	0
1101	0
1110	1
1111	0

980

1's	Terms	Pass 1	Pass 2
0	0000		
1	0001		
	1000		
2	0110		
	1001		
	1010		
3	1011		
	1110		

982

1's	Terms	Pass 1	Pass 2
0	0000 ✓	000x ✓	x00x
1	0001 ✓	x000 ✓	x0x0
	1000 ✓	x001 ✓	10xx
2	0110 ✓	100x ✓	
	1001 ✓	10x0 ✓	
	1010 ✓	10x1 ✓	
3	1011 ✓	101x ✓	
	1110 ✓	x110	
		1x10	

984

986

	0110	1010	1011	1110
x110 *	✓			✓
1x10		✓		✓
x00x				
x0x0		✓		
10xx *		✓	✓	

990

$$f(A,B,D,C) = \sum(0110,1010,1011,1110) + \sum_{d.c.}(0000,0001,1000,1001)$$
$$= \sum(x110,10xx)$$
$$= BC\overline{D} + A\overline{B}$$

Figure 11: Example of Quine-McCluskey Method for Logic Minimisation With "Don't Cares" (Prior Art).